

# WSCA Summer Assignment

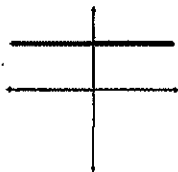
## AP Calculus

# Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator.

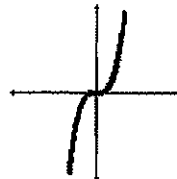
**Constant**

$$f(x) = a$$



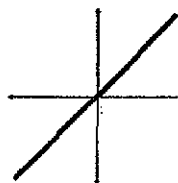
**Cubic**

$$f(x) = x^3$$



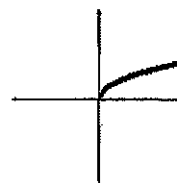
**Identity**

$$f(x) = x$$



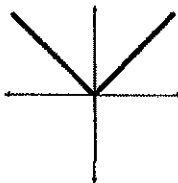
**Square Root**

$$f(x) = \sqrt{x}$$



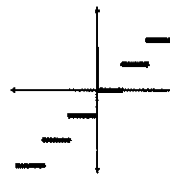
**Absolute Value**

$$f(x) = |x|$$



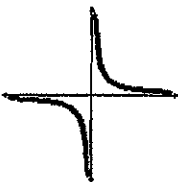
**Greatest Integer**

$$f(x) = [x]$$



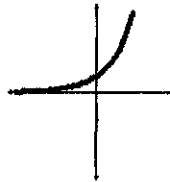
**Reciprocal**

$$f(x) = \frac{1}{x}$$



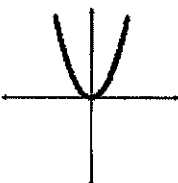
**Exponential**

$$f(x) = a^x$$



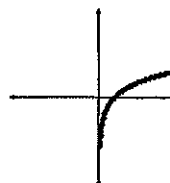
**Quadratic**

$$f(x) = x^2$$



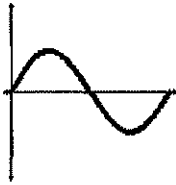
**Logarithmic**

$$f(x) = \ln x$$

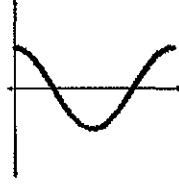


### Trig Functions

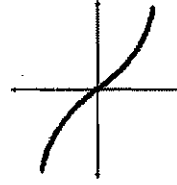
$$f(x) = \sin x$$



$$f(x) = \cos x$$



$$f(x) = \tan x$$



### Polynomial Functions:

A function  $P$  is called a polynomial if  $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$   
Where  $n$  is a nonnegative integer and the numbers  $a_0, a_1, a_2, \dots, a_n$  are constants.

Even degree

Odd degree

Leading coefficient sign

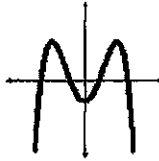
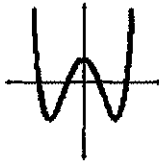
Leading coefficient sign

Positive

Negative

Positive

Negative



- Number of roots equals the degree of the polynomial.
- Number of  $x$  intercepts is less than or equal to the degree.
- Number of "bends" is less than or equal to (degree - 1).

## Formulas and Identities

### Trig Formulas:

Arc Length of a circle:  $L = r\theta$  or  $L = \frac{d}{360} \cdot 2\pi r$

Area of a sector of a circle:  $\text{Area} = \frac{1}{2}r^2\theta$  or  $\text{Area} = \frac{d}{360} \cdot \pi r^2$

### Solving parts of a triangle:

Law of Sines:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines:  $a^2 = b^2 + c^2 - 2bc \cos A$   
 $b^2 = a^2 + c^2 - 2ac \cos B$   
 $c^2 = a^2 + b^2 - 2ab \cos C$

### Area of a Triangle:

$\text{Area} = \frac{1}{2}bc \sin A$  or  $\text{Area} = \frac{1}{2}ac \sin B$  or  $\text{Area} = \frac{1}{2}ab \sin C$

Hero's formula:  $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ , where  $s =$  semi perimeter

### Ambiguous Case:

$\theta$  is acute

Compute:  $\text{alt} = \text{adj} \cdot \sin \theta$

$\text{opp} < \text{alt}$  No triangle  
 $\text{opp} = \text{alt}$  1 triangle (right)  
 $\text{opp} > \text{alt}$  1 triangle

$\text{alt} < \text{opp} < \text{adj}$  2 triangles

$\theta$  is obtuse or right

$\text{opp} \leq \text{adj}$  No triangle  
 $\text{opp} > \text{adj}$  1 triangle

Does a triangle exist? Yes - when

$(\text{difference of 2 sides}) < (\text{third side}) < (\text{Sum of 2 sides})$

## Formulas and Identities. continued

### Trig Identities:

#### Reciprocal Identities:

$$\csc A = \frac{1}{\sin A} \quad \sec A = \frac{1}{\cos A} \quad \cot A = \frac{1}{\tan A}$$

#### Quotient Identities:

$$\tan A = \frac{\sin A}{\cos A} \quad \cot A = \frac{\cos A}{\sin A}$$

#### Pythagorean Identities:

$$\sin^2 A + \cos^2 A = 1 \quad \tan^2 A + 1 = \sec^2 A \quad 1 + \cot^2 A = \csc^2 A$$

#### Sum and Difference Identities:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \quad \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \quad \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

#### Double Angle Identities:

$$\sin(2A) = 2\sin A \cos A \quad \tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos(2A) = \cos^2 A - \sin^2 A \quad \cos(2A) = 2\cos^2 A - 1 \quad \cos(2A) = 1 - 2\sin^2 A$$

#### Half Angle Identities:

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}} \quad \cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} \quad \tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

#### Polar Formulas:

$$x^2 + y^2 = r^2 \quad x = r \cos \theta \quad y = r \sin \theta \quad \tan^{-1} \frac{y}{x} = \theta \quad x > 0, \quad \tan^{-1} \frac{y}{x} = \theta + \pi \quad x < 0$$

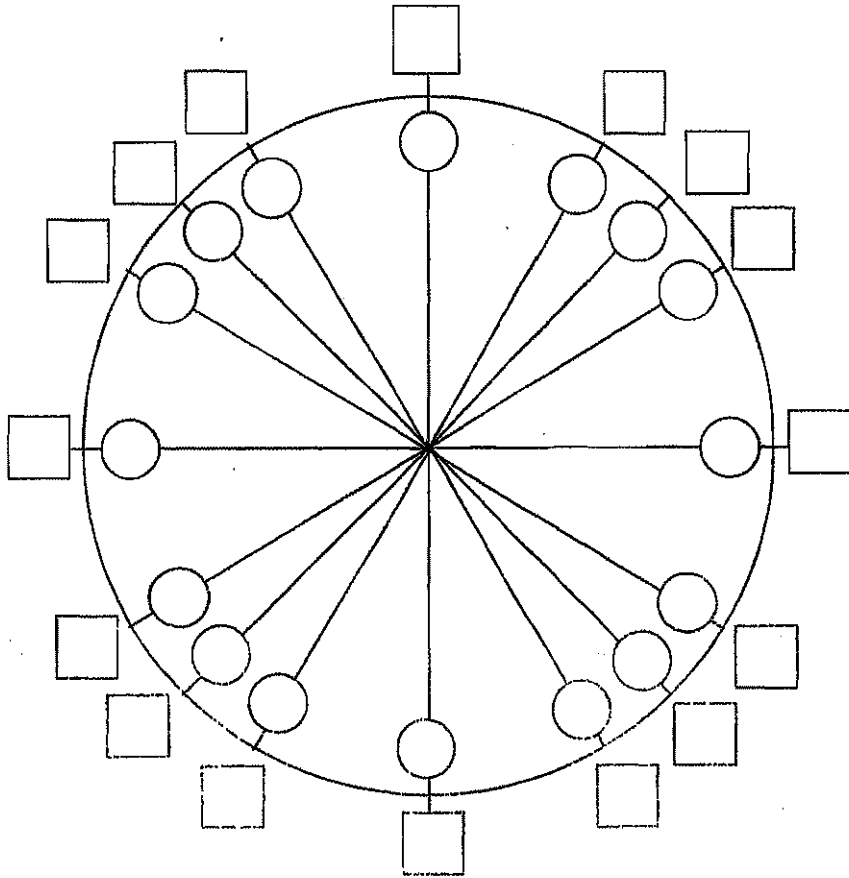
#### Geometric Formulas:

$$\text{Area of a trapezoid: } A = \frac{1}{2}h(b_1 + b_2) \quad \text{Area of a triangle: } A = \frac{1}{2}bh$$

$$\text{Area of an equilateral triangle: } A = \frac{\sqrt{3}}{4}s^2$$

$$\text{Area of a circle: } A = \pi r^2 \quad \text{Circumference of a circle: } C = 2\pi r \text{ or } C = d\pi$$

# Unit Circle – Degrees and Radians



Place degree measures in the circles.

Place radian measure in the squares.

Place  $(\cos \theta, \sin \theta)$  in parenthesis outside the square.

Place  $\tan \theta$  outside the parenthesis.

$\tan \theta =$  \_\_\_\_\_

$\cot \theta =$  \_\_\_\_\_

$\csc \theta =$  \_\_\_\_\_

$\sec \theta =$  \_\_\_\_\_

## SKILLS NEEDED FOR CALCULUS

### I. Algebra:

- \*A. Exponents (operations with integer, fractional, and negative exponents)
- \*B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
- C. Rationalizing (numerator and denominator)
- \*D. Simplifying rational expressions
- \*E. Solving algebraic equations and inequalities (linear, quadratic, higher order using synthetic division, rational, radical, and absolute value equations)
- F. Simultaneous equations - **Systems**

### II. Graphing and Functions

- \*A. Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
- B. Conic Sections (circle, parabola, ellipse, and hyperbola)
- \*C. Functions (definition, notation, domain, range, inverse, composition)
- \*D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric, piece-wise, inverse functions)
- E. Tests for symmetry: odd, even

### III. Geometry

- A. Pythagorean Theorem
- B. Area Formulas (Circle, polygons, surface area of solids)
- C. Volume formulas
- D. Similar Triangles

### \* IV. Logarithmic and Exponential Functions

- \*A. Simplify Expressions (Use laws of logarithms and exponents)
- \*B. Solve exponential and logarithmic equations (include ln as well as log)
- \*C. Sketch graphs
- \*D. Inverses

### \* V. Trigonometry

- \*\*A. Unit Circle (definition of functions, angles in radians and degrees)
- B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
- \*C. Solve equations
- \*D. Inverse Trigonometric functions
- E. Right triangle trigonometry
- \*F. Graphs

### ~~VI. Limits~~

- ~~A. Concept of a limit~~
- ~~B. Find limits as  $x$  approaches a number and as  $x$  approaches  $\infty$~~

\* A solid working foundation in these areas is very important.

## Calculus Prerequisite Problems

Work the following problems on your own paper. Show all necessary work.

### I. Algebra

A. Exponents: 1)  $\frac{(8x^3yz)^{1/3}(2x)^3}{4x^{1/3}(yz^{2/3})^{-1}}$

B. Factor Completely:

2)  $9x^2 + 3x - 3xy - y$  (use grouping)    3)  $64x^6 - 1$  *Hint: Factor as difference of squares first, then as the sum and difference of cubes second.*

4)  $42x^4 + 35x^2 - 28$     5)  $15x^{5/2} - 2x^{3/2} - 24x^{1/2}$  *Hint: Factor GCF  $x^{1/2}$  first.*

6)  $x^{-1} - 3x^{-2} + 2x^{-3}$  *Hint: Factor out GCF  $x^{-3}$  first.*

C. Rationalize denominator/ numerator:

7)  $\frac{3-x}{1-\sqrt{x-2}}$     8)  $\frac{\sqrt{x+1}+1}{x}$

D. Simplify the rational expression:

9)  $\frac{(x+1)^3(x-2) + 3(x+1)^2}{(x+1)^4}$

E. Solve algebraic equations and inequalities

10. - 11. Use synthetic division to help factor the following, state all factors and roots.

10)  $p(x) = x^3 + 4x^2 + x - 6$

11)  $p(x) = 6x^3 - 17x^2 - 16x + 7$

~~X~~ Explain why  $\frac{3}{2}$  cannot be a root of  $f(x) = 4x^5 + cx^3 - dx + 5$ , where c and d are integers.

(hint: You can look at the possible rational roots.)

~~X~~ Explain why  $f(x) = x^4 + 7x^2 + x - 5$  must have a root in the interval  $[0, 1]$ , ( $0 \leq x \leq 1$ )  
Check the graph and use signs of  $f(0)$  and  $f(1)$  to justify your answer.

Solve: You may use your graphing calculator to check solutions. Write your ans. in interval notation.

14)  $(x+3)^2 > 4$     ~~X~~  $\frac{x+5}{x-3} \leq 0$     16)  $3x^3 - 14x^2 - 5x \leq 0$  (Factor first)

17)  $x < \frac{1}{x}$     18)  $\frac{x^2-9}{x+1} \geq 0$     19)  $\frac{1}{x-1} + \frac{4}{x-6} > 0$

20)  $x^2 < 4$     21)  $|2x+1| < \frac{1}{4}$



F. *Solve the system.* Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

$$\begin{aligned} 22) \quad x - y + 1 &= 0 \\ y - x^2 &= -5 \end{aligned}$$

$$\begin{aligned} 23) \quad x^2 - 4x + 3 &= y \\ -x^2 + 6x - 9 &= y \end{aligned}$$

## II. Graphing and Functions:

A. *Linear graphs:* Write the equation of the line described below.

24) Passes through the point (2, -1) and has slope  $-\frac{1}{3}$ .

25) Passes through the point (4, -3) and is perpendicular to  $3x + 2y = 4$ .

26) Passes through (-1, -2) and is parallel to  $y = \frac{3}{5}x - 1$ .

~~X~~ *Conic Sections:* Write the equation in standard form and identify the conic.

~~X~~  $x = 4y^2 + 8y - 3$

~~X~~  $4x^2 - 16x + 3y^2 + 24y + 52 = 0$

C. *Functions:* Find the domain and range of the following.

Note: domain restrictions - denominator  $\neq 0$ , argument of a log or  $\ln > 0$ ,  
radicand of even index must be  $\geq 0$   
range restrictions- reasoning, if all else fails, use graphing calculator

29)  $y = \frac{3}{x-2}$

30)  $y = \log(x-3)$

31)  $y = x^4 + x^2 + 2$

32)  $y = \sqrt{2x-3}$

33)  $y = |x-5|$

34) domain only:  $y = \frac{\sqrt{x+1}}{x^2-1}$

35) Given  $f(x)$  below, graph over the domain  $[-3, 3]$ , what is the range?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x-2 & \text{if } x < -1 \end{cases}$$

Find the composition/inverses as indicated below.

Let  $f(x) = x^2 + 3x - 2$     $g(x) = 4x - 3$     $h(x) = \ln x$     $w(x) = \sqrt{x-4}$

36)  $g^{-1}(x)$    37)  $h^{-1}(x)$    38)  $w^{-1}(x)$ , for  $x \geq 4$    39)  $f(g(x))$    40)  $h(g(f(1)))$

41) Does  $y = 3x^2 - 9$  have an inverse function? Explain your answer.

Let  $f(x) = 2x$ ,  $g(x) = -x$ , and  $h(x) = 4$ , find

~~42)~~  $(f \circ g)(x)$    ~~43)~~  $(f \circ g \circ h)(x)$

~~44)~~ Let  $s(x) = \sqrt{4-x}$  and  $t(x) = x^2$ , find the domain and range of  $(s \circ t)(x)$ .

**D. Basic Shapes of Curves:**

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

45)  $y = \sqrt{x}$    46)  $y = \ln x$    47)  $y = \frac{1}{x}$    48)  $y = |x - 2|$

49)  $y = \frac{1}{x-2}$    50)  $y = \frac{x}{x^2-4}$    51)  $y = 2^{-x}$    52)  $y = 3 \sin 2(x - \frac{\pi}{6})$

$$53) f(x) = \begin{cases} \sqrt{25-x^2} & \text{if } x < 0 \\ \frac{x^2-25}{x-5} & \text{if } x \geq 0, x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$$

**E. Even, Odd, Tests for Symmetry:**

Identify as odd, even, or neither and justify you're answer. To justify your answer you must show substitution using  $-x$ ! It is not enough to simply check a number.

Even: $f(x) = f(-x)$	Odd: $f(-x) = -f(x)$
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54)  $f(x) = x^3 + 3x$    55)  $f(x) = x^4 - 6x^2 + 3$    56)  $f(x) = \frac{x^3 - x}{x^2}$

57)  $f(x) = \sin 2x$    58)  $f(x) = x^2 + x$    59)  $f(x) = x(x^2 - 1)$

60)  $f(x) = \frac{1 + |x|}{x^2}$

61) What type of function (even or odd) results from the product of two  
         even functions?          odd functions?

**Test for symmetry. Show substitution with variables to justify your answer.**

Symmetric to y axis: replace x with -x and relation remains the same.

Symmetric to x axis: replace y with -y and relation remains the same.

Origin symmetry: replace x with -x, y with -y and the relation is equivalent.

62)  $y = x^4 + x^2$     63)  $y = \sin(x)$     64)  $y = \cos(x)$

65)  $x = y^2 + 1$     66)  $y = \frac{|x|}{x^2 + 1}$

#### IV LOGARITHMIC AND EXPONENTIAL FUNCTIONS

**A. Simplify Expressions:**

~~67)~~  $\log_4\left(\frac{1}{16}\right)$     68)  $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$     69)  $\log_9 27$

70)  $\log_{125}\left(\frac{1}{5}\right)$     71)  $\log_w w^{45}$     72)  $\ln e$     73)  $\ln 1$     74)  $\ln e^2$

**B. Solve equations:**

75)  $\log_6(x+3) + \log_6(x+4) = 1$     76)  $\log x^2 - \log 100 = \log 1$     77)  $3^{x+1} = 15$

#### V TRIGONOMETRY

**A. Unit Circle:** Know the unit circle - radian and degree measure. Be prepared for a quiz.

78) State the domain, range and fundamental period for each function?

a)  $y = \sin x$     b)  $y = \cos x$     c)  $y = \tan x$

**B. Identities:**

Simplify:    79)  $\frac{(\tan^2 x)(\csc^2 x) - 1}{(\csc x)(\tan^2 x)(\sin x)}$     80)  $1 - \cos^2 x$     81)  $\sec^2 x - \tan^2 x$

82) Verify:  $(1 - \sin^2 x)(1 + \tan^2 x) = 1$

**C. Solve the Equations**

83)  $\cos^2 x = \cos x + 2$ ,  $0 \leq x \leq 2\pi$     84)  $2 \sin(2x) = \sqrt{3}$ ,  $0 \leq x \leq 2\pi$

85)  $\cos^2 x + \sin x + 1 = 0$ ,  $0 \leq x \leq 2\pi$

D. Inverse Trig Functions: Note:  $\sin^{-1} x = \text{Arcsin } x$

86)  $\text{Arcsin } 1$

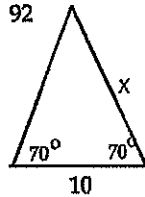
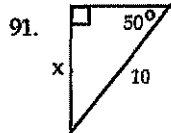
87)  $\text{Arcsin} \left( -\frac{\sqrt{2}}{2} \right)$

88)  $\text{Arccos} \left( \frac{\sqrt{3}}{2} \right)$

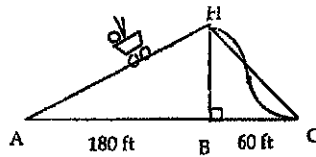
89)  $\sin \left( \text{Arccos} \left( \frac{\sqrt{3}}{2} \right) \right)$

90. State domain and range for:  $\text{Arcsin}(x)$ ,  $\text{Arccos}(x)$ ,  $\text{Arctan}(x)$

E. Right Triangle Trig: Find the value of  $x$ . (Note: Degree measure!)



93.



93) The roller coaster car shown in the diagram above takes 23.5 sec. to go up the 23 degree incline segment AH and only 2.8 seconds to go down the drop from H to C. The car covers horizontal distances of 180 feet on the incline and 60 feet on the drop. Decimals in answer may vary.

- How high is the roller coaster above point B?
- Find the distances AH and HC.
- How fast (in ft/sec) does the car go up the incline?
- What is the approximate average speed of the car as it goes down the drop?
- Assume the car travels along HC. Is your approximate answer too big or too small?

(Advanced Mathematics, Richard G. Brown, Houghton Mifflin, 1994, pg 336)

~~X~~ **Graphs:** Identify the amplitude, period, horizontal, and vertical shifts of these functions.

~~X~~  $y = -2\sin(2x)$

~~X~~  $y = -\pi \cos\left(\frac{\pi}{2}x + \pi\right)$

**G. Be able to do the following on your graphing calculator:**

Be familiar with the **CALC** commands: value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find the information. Answers should be accurate to 3 decimal places. Sketch graph.

96. - 99. Given the following function  $f(x) = 2x^4 - 11x^3 - x^2 + 30x$ .

96. Find all roots. Note: Window x min: -10 x max: 10 scale 1  
y min: -100 y max: 60 scale 0

97. Find all local maxima.

98. Find all local minima.

*A local maximum or local minimum is a point on the graph where there is a highest or lowest point within an interval such as the vertex of a parabola.*

99. Find the following values:  $f(-1)$ ,  $f(2)$ ,  $f(0)$ ,  $f(.125)$

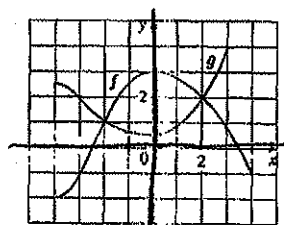
100. Graph the following two functions and find their points of intersection using the intersect command on your calculator.

$y = x^3 + 5x^2 - 7x + 2$  and  $y = .2x^2 + 10$  Window: x min: -10 x max: 10 scale 1  
y min: -10 y max: 50 scale 0

**VI. Functions and Models**

101. The graphs of  $f$  and  $g$  are given.

- (a) State the values of  $f(-4)$  and  $g(3)$ .
- (b) For what values of  $x$  if  $f(x)=g(x)$ ?
- (c) Estimate the solution of the equation
- (d) On what interval is  $f$  decreasing?
- (e) State the domain and range of  $f$ .
- (f) State the domain and range of  $g$ .



$f(x) = -1$ .

102. The number  $N$  (in thousands) of cellular phone subscribers in Malaysia is shown in the table. (Midyear estimates are given.)

t	1991	1993	1995	1997
N	132	304	873	2461

- (a) Use the data to sketch a rough graph of  $N$  as a function of  $t$ .
- (b) Use your graph to estimate the number of cell-phone subscribers in Malaysia at midyear in 1994 and 1996.

103. If  $f(x) = 3x^2 - x + 2$ , find  $f(2)$ ,  $f(-2)$ ,  $f(a)$ ,  $f(-a)$ ,  $f(a+1)$ ,  $2f(a)$ ,  $f(a^2)$ ,  $[f(a)]^2$ , and  $f(a+h)$ .

104. Find the domain of each function.

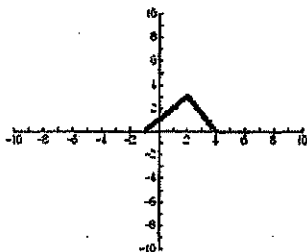
a)  $f(x) = \frac{x}{3x-1}$

b)  $g(u) = \sqrt{u} + \sqrt{4-u}$

~~105.~~ Find an expression for the bottom half of the parabola  $x+(y-1)^2=0$ .

106. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

107. Find an expression for the function whose graph is the given curve. - Hint: It is piecewise



~~108.~~ Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 113 chirps per minute at  $70^\circ\text{F}$  and 173 chirps per minute at  $80^\circ\text{F}$ .

(a) Find a linear equation that models the temperature  $T$  as a function of the number of chirps per minute  $N$ .

(b) What is the slope of the graph? What does it represent?

(c) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

109. At the surface of the ocean, the water pressure is the same as the air pressure above the water,  $15\text{ lb/in}^2$ . Below the surface, the water pressure increases by  $4.34\text{ lb/in}^2$  for every 10 ft of descent.

(a) Express the water pressure as a function of the depth below the ocean surface.

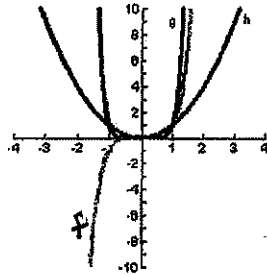
(b) At what depth is the pressure  $100\text{ lb/in}^2$ ?

110. Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, or logarithmic function.

(a)  $f(x) = \sqrt[3]{x}$     (b)  $g(x) = \sqrt{1-x^2}$     (c)  $h(x) = x^9 + x^4$     (d)  $r(x) = \frac{x^3 + 1}{x^3 + x}$

111. Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator).

(a)  $y = x^2$     (b)  $y = x^5$     (c)  $y = x^3$

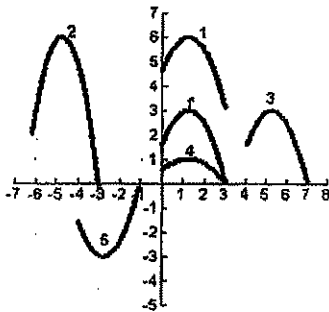


112. Suppose the graph of  $f$  is given. Write equations for the graphs that are obtained from the graph of  $f$  as follows.

- |  |   |
|--|---|
| (a) Shift 3 units upward.                | (b) Shift 3 units downward.             |
| (c) Shift 3 units to the right.          | (d) Shift 3 units to the left.          |
| (e) Reflect about the x-axis.            | (f) Reflect about the y-axis.           |
| (g) Stretch vertically by a factor of 3. | (h) Shrink vertically by a factor of 3. |

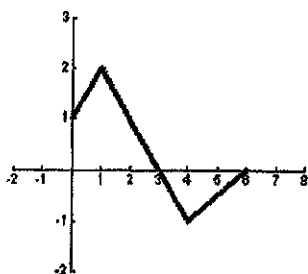
113. The graph of  $y = f(x)$  is given. Match each equation with its graph and give reasons for your choices.

(a)  $y = f(x-4)$     (b)  $y = f(x)+3$     (c)  $y = 1/3 f(x)$     (d)  $y = -f(x+4)$     (e)  $y = 2f(x+6)$



114. The graph of  $f$  is given. Use it to graph the following functions.

- (a)  $y = f(2x)$       (b)  $y = f(\frac{1}{2}x)$       (c)  $y = f(-x)$       (d)  $y = -f(-x)$



~~115~~ Graph the following, not by plotting points, but by starting with the graph of one of the standard functions and applying the appropriate transformations.

$$y = \frac{1}{3} \sin\left(x - \frac{\pi}{6}\right)$$

~~116~~ Find  $f+g$ ,  $f-g$ ,  $fg$ , and  $f/g$  and state their domains.

$$f(x) = x^3 + 2x^2, \quad g(x) = 3x^2 - 1$$

~~117~~ Find the functions  $f \circ g$ ,  $g \circ f$ ,  $f \circ f$ , and  $g \circ g$  and their domains.

$$f(x) = \sin x, \quad g(x) = 1 - \sqrt{x}$$

118. Express the function in the form  $f \circ g$ .

$$F(x) = (x^2 + 1)^{10}$$

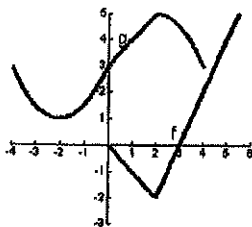
119. Use a graphing calculator to determine which of the given viewing rectangles produces the most appropriate graph of the function  $f(x) = 10 + 25x - x^3$ .

- (a)  $[-4, 4]$  by  $[-4, 4]$   
 (b)  $[-10, 10]$  by  $[-10, 10]$   
 (c)  $[-20, 20]$  by  $[-100, 100]$   
 (d)  $[-100, 100]$  by  $[-200, 200]$



120. Use the given graphs of  $f$  and  $g$  to evaluate each expression, or explain why it is undefined.

- (a)  $f(g(2))$     (b)  $g(f(0))$   
 (c)  $(f \circ g)(0)$     (d)  $(g \circ f)(6)$     (e)  $(g \circ g)(-2)$     (f)  $(f \circ f)(4)$



**121 and 122. Determine an appropriate viewing rectangle for the given function and use it to draw each graph.**

121.  $f(x) = 5 + 20x - x^2$

122.  $f(x) = \sqrt[4]{81 - x^4}$

~~123~~ Graph the ellipse  $4x^2 + 2y^2 = 1$  by graphing the functions whose graphs are the upper and lower halves of the ellipse.

124. Find all solutions of the equation correct to three decimal places.

$$x^3 - 9x^2 - 4 = 0$$

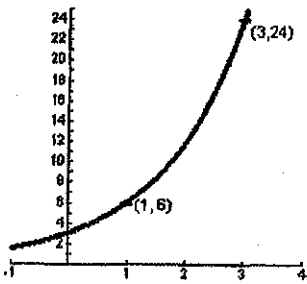
~~125~~ Graph the given functions on a common screen. How are these graphs related?

$$y = 2^x, \quad y = e^x, \quad y = 5^x, \quad y = 20^x$$

126. Starting with the graph of  $y = e^x$ , write the equation of the graph that results from

- shifting 2 units downward
- shifting 2 units to the right
- reflecting about the  $x$ -axis
- reflecting about the  $y$ -axis
- reflecting about the  $x$ -axis and then about the  $y$ -axis

127. Find the exponential function  $f(x) = Ca^x$  whose graph is given.



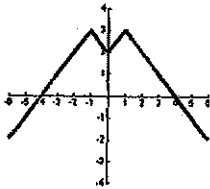
128. Under ideal conditions a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria. — Exponential Growth.

- What is the size of the population after 15 hours?
- What is the size of the population after  $t$  hours.
- Estimate the size of the population after 20 hours.
- Graph the population function and estimate the time for the population to reach 50,000.

129. Determine whether this function is one-to-one.

$x$	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

130. Determine whether this function is one-to-one.



131. Here is a verbal description of a function. Determine whether this function is one-to-one.  $F(t)$  is the height of a football  $t$  seconds after kickoff.

**For #132-134, find a formula for the inverse of the function.**

132.  $f(x) = \sqrt{10-3x}$

133.  $f(x) = e^{x^2}$

134.  $y = \ln(x+3)$

**For #135-136, find the exact value of each expression (non-calculator).**

135. (a)  $\log_2 64$       (b)  $\log_6 \frac{1}{36}$

136. (a)  $\log_{10} 1.25 + \log_{10} 80$       (b)  $\log_5 10 + \log_5 20 - 3\log_5 2$

137. Express the given quantity as a single logarithm.  
 $2 \ln 4 - \ln 2$

138. The graph shown gives a salesman's distance from his home as a function of time on a certain day. Describe in words what the graph indicates about his travels on this day.

